Finite Elements Adding and Removing Method for Two-Dimensional Shape Optimal Design

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A simple procedure to add and remove material simultaneously along the boundary is developed to optimize the shape of a two dimensional elastic problems and to minimize the maximum von Mises stress. The results for the two dimensional infinite plate with a hole, are close to the theoretical results of an elliptical boundary and the stress concentration is reduced by half for the fillet problem. The proposed shape optimization method, when compared with existing derivative based shape optimization methods has many features such as simplicity, applicability, flexiblity, computational efficiency and a much better control on stresses on the design boundary.

Key Words: Shape Optimization, Self-Designing Structures, Finite Element Analysis, Boundary Smoothing, Evolution, Topology, Hole in an Infinite Plate, Fillet

1. Introduction

In an optimal shape design, the mathematical programming method have conventionally been used to the find optimal geometry of a structure for the given design constraints. The usual approach is to represent the whole design boundary in terms of one flexible curve such as cubic spline and b-spline curves(Shyy and Fleury, 1988). This method is based on the initial design variable of the boundary, creating in the process of list of new variables for each iteration, whose values are used to indicate the points in space through which the boundary of the structure passes.

In the traditional optimization technique, such as mathematical programming and optimality criteria methods, the derivatives of the objective and constraint functions with respect to design variables are needed. So such methods can handle only problems with continuous design variables. Also the recreation of finite element mesh is necessary for each design variable change in each iteration.

The evolutionary optimization technique offers much more direct approach in finding the optimal layouts(Xie and Steven, 1993, 1997; Hinton and Sienz, 1995). Their method is based on the concept of gradually removing redundant material to achieve the optimal design of a structure. The stress distribution within the the structure is found using the finite element analysis[FEA], and the material usage in the structure may not be effective in terms of stress distribution. This method has been applied successfully in many structural topology optimization problems. After the FEA analysis the maximum von Mises stress within the structure is multiplied by a rejection ratio, elements with a stress below the factored maximum are effectively removed, either by

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removing totally the elements from the analysis, or by reducing the element's Young's modulus value to almost zero. The resulting structure is then reanalysed, and the same removal criteria are reapplied to remove more elements. This process continues until a steady state of element removal is archived and eventually the majority of the structure becomes fully stressed. The simplicity of the method and the easy implementation of the algorithm is a major advantage of the method. The main disadvantage of the material removal only method is that it is impossible to find an optimal shape outside the predetermined area. Therefore a larger area is needed at the start of this method.

Recently a new structural optimization approachs called Reverse Adaptivity(RA) and Self Designing Structure(SDS) have been developed. In the RA method(Reynold et al, 1999), after the initial finite element is defined the method proceeds with refinement of low stressed regions of finite element mesh by element subdivision. Following this any low stressed elements are removed and the process is repeated. The SDS method(Christie et al, 1998; Bull et al, 1999; Bull and Lim, 1999) not only removes materials from low stressed areas but also is capable of adding materials in highly stressed areas to lower the high stresses. In this semi-automatic method, the designer manually redefines the boundary of the structure along a chosen contour and remeshes it for each iteration. This method will find a topology of a structure with fully stressed in the entire domain. The SDS Method has been successfully applied to give the optimal topology of a structure for given boundary conditions and applied forces.

In this paper the, the SDS approach is used with the simultaneous addition and removal of material along the boundary to find an optimal shape of the boundary within the desired stress range. The SDS method has significant advantages over existing shape optimization approaches. The most important point is the simplicity of the method. The mathematical problem formulation for optimization which involves an objective function, constraints and design variables are not required in the SDS method. Since the SDS method does not use the conventional derivative based optimization algorithm, it is simple to understand and implement in a computer program. Another important advantage of the SDS method is simplicity of geometry definition for every iteration. Since the SDS method deals with element removal and addition the meshing remains the same except where the addition and removal took place, it does not need to remesh the whole domain for each iteration as in conventional derivative based optimization methods.

It was found that the SDS method used in this paper is simple and can be easily implemented with into general FEA program. The method was implemented using PAFEC(PAFEC Limited, 1994).

2. Shape Optimization Procedure with Finite Elements Adding Removing Method

A simple iterative procedure has been developed to archive the optimal shape of a 2-D elastic boundary to minimize the maximum von Mises stress along the design boundary. The developed fully automatic material adding and removing method has advantage that it does not require the decision concerns for the optimal shape of the boundary in design process. The procedure consist of following major iterative steps.

(1) Read in the FE model data and apply the boundary condition;
(2) Refine boundary elements and perform FE analysis;
(3) Add or remove elements depending on the magnitude of the average nodal von Mises stress along the boundary;
(4) Smooth the design boundary after adding and removing elements;
(5) Write data to the file for the next iteration and return to step (1):

The procedure iterates through all steps until all the nodal von Mises stresses along the

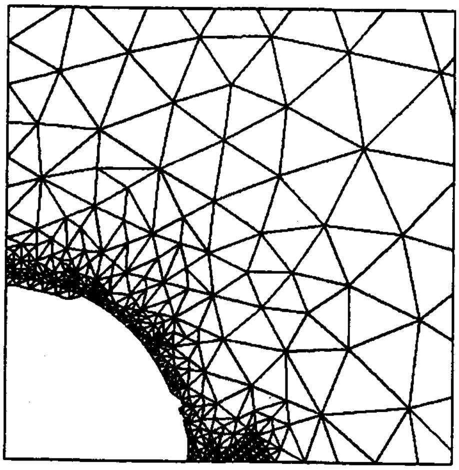


Fig. 1 Design boundary refinement

boundary remain within the desired design range. The desired stress could be the maximum allowable stress or the endurance limit for a fatigue analysis.

2.1 Read in the FE model data and apply the boundary conditions

The analysis model data, the boundary condition data and the elements mesh data are read in for the finite analysis. The data file is prepared to interface with the commercial FEA package PAFEC. Using the PAFEC, the initial mesh with load and boundary conditions are defined, and after analysis the post-processing is carried out with the package. This enables the user to incorporate many well developed features in the commercial software.

2.2 Refine boundary elements and FE analysis

The stresses along the design boundary need more accurate stress analysis than inside of the structure because the addition and removal will take place along the design boundary. After reading in FE model data, the elements along the design boundary are subdivided into smaller elements by the element bisection technique in conventional elements adaptivity as shown in Fig. 1. The size of subdivided elements can be adjusted possible by controlling the final refined element edge length parameter. This refinement enables the better stress results to be obtained but also to control the amount of adding and removal material, by controlling the refinement element size. The stress analysis of two dimensional structure is performed using a 6 noded triangle elements. The finite element model with finer boundary elements is solved and the nodal stress returned. The stresses are averaged to obtain the von Mises stress in each elements in the structure.

2.3 Elements addition and removal

It could be said that one of the most common purposes of structural optimization is to minimize the weight of the structure while satisfying the stress constraints. The minimum weight can be achieved by removing the elements along the design boundary where the von Mises stress is smaller than the prescribed cut off stress, and stress can be lowered by adding elements at the node along the design boundary which have greater von Mises stresses. In order to decide addition and removal of elements, two stress values must be set: the maximum stress threshold and the removal stress threshold. The maximum stress threshold is a fixed limit which stress along the design boundary should not exceed. If the average von Mises stress at a boundary node is greater than the maximum stress threshold stress then add one or two element depending on the angle of the two adjacent edges at the node. The removal stress threshold is a fixed limit which, ideally, all stress along the boundary should greater than this threshold. If the average von Mises stress at a boundary node is smaller than this prescribed stress then remove all the elements containing that node. The addition and removal procedures are as follow:

1) Identify the design boundary edges and nodes, store for later use. The boundary edges can be easily identified by checking if the edge share with other elements or not.
2) Calculate the average nodal stress from all the elements containing the node along the design boundary. The average element stresses from FE analysis are used to obtain an average von Mises stress for each node along the boundary.

The nodal average von Mises stress are calculated by averaging the elements stress at the element which contain the node along the node.

3) For each node decide whether to add or remove elements or move to next node depending on the magnitude of the average nodal stress. The elements are added or removed from the physical model, and update the FEA data such as nodal position and element connectivity. If the angle between two adjacent edges is concave and less than 120 degree then add an element with 3 existing nodes in sequence which has two adjacent edges. If the angle between two adjacent edge is greater than 120 degree then a new node is generated which divides the angle in half at the distance of the average edge length of the adjacent two edges. Two elements are generated with a new node and 3 existing nodes, and the generated new elements data are added to the FEA data base. The scalar and vector products of unit vectors of the edges were used to determine the angle of the adjacent edges. The removal of elements can be done easily by just removal of element's data from the FEA data base. The proposed method requires only one FEA per each iteration as the adding and removal process takes place on the boundary node sequentially. This is one of the main advantages of the method. In Figs. 1 and 2, the elements addition and removal can be seen along the initial a quarter circle boundary line. As shown in the figures, materials are added in the high stressed region and materials are removed from the low stressed region.

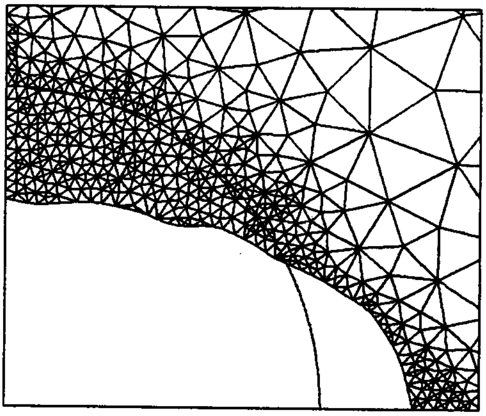


Fig. 2 Adding and removing of elements along the boundary with initial circular boundary

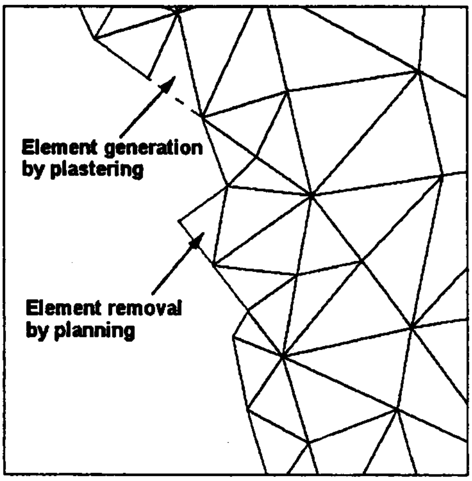


Fig. 3 Boundary smoothing with plastering and planning

2.4 Smoothing the boundary

The shape of the boundary changes for each iteration, and the stress distribution along the boundary is decided by the shape of the boundary. The shape of the boundary is the most critical factor for the accurate stress analysis along the boundary. The shape of the boundary line changes to a saw-tooth type shape after elements adding and removing procedure at a node for each iteration. Figure 3 shows a typical saw-tooth shape of boundary line during iteration without boundary smoothing. This saw-tooth type shape along the boundary yields local stress concentrations and uneven stress distribution which yields inaccurate finite element analysis. Therefore it is necessary that a smooth boundary must be kept throughout the iteration. Figure 2 also shows the typical smoothed boundary curve during the iteration. The boundary smoothing procedure has the following steps.

1) Plastering and planning the boundary

Figure 3 shows the plastering and planning operation for boundary smoothing. If the angle between the adjacent edge is concave and acute then an element with 3 nodes is generated. This is similar to filling the gap of crack. The boundary planning is done by cut off the element, if the boundary shape at a node is projected out from the boundary curve.

2) Averaging the boundary node

The position of nodes along the boundary can be modified by averaging the adjacent two node. This simple reposition of nodes can smoothing the peaks and valleys effectively along the boundary.

3) Smoothing by bspline curve

Final smoothing of the boundary curve is done by a bspline curve which goes through all the nodes along the boundary.

3. Examples of the Application of the Method

Two classical boundary shape optimal design problems are used to demonstrate the effectiveness of the proposed shape optimal design approach. A plate problem with a central hole and a fillet problem are the examples used for minimize the maximum stress along the design boundary.

3.1 Example 1: Optimization of a plate with a hole in biaxial tension

A classical example problem in shape optimization is to find the optimal shape of a hole in an infinite isotropic homogeneous elastic plate under biaxial traction(Shyy and Fleury, 1988; Kim and Kwak, 1996). Because of symmetry, a quarter of the plate is modeled as shown in Fig. 1. In conventional parametric shape optimization design of this example, the design variables have some restraints such as direction of movements and tangency requirements at the end of the curve. In the proposed approach there are no design variables, and no constraints on the design variables as in the conventional shape optimization design problems. The problem is to minimize the maximum von Mises stress in the plate along the boundary. The theoretical solution of the hole shape which minimize the maximum von Mises stress along the hole

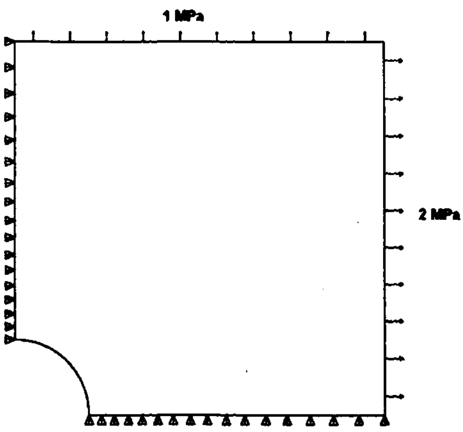


Fig. 4 Initial quarter model with a circular hole

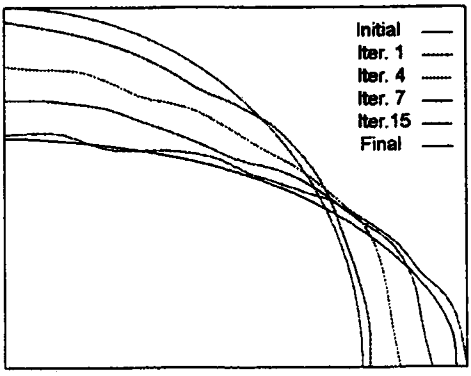


Fig. 5 Iteration history of hole shape

boundary is an ellipse with an axis ratio of the same as the traction ratio in the orthogonal direction(Wheeler and Kunin, 1982). In an infinite biaxial tension field, the stress along the ellipse boundary is always the same regardless of the size of the ellipse thus the solution of the problem is an ellipse of undetermined size.

For each iteration, the von Mises stresses are calculated along the boundary, and at each node to find the average stress of each element which contains the node along the boundary curve. The applied biaxial tension is 2 MPa in x-direction and 1 MPa in y-direction as shown in Fig. 4. The von Mises stress along the ellipse boundary is 3 MPa for the give biaxial tension field. If the stress at a node along the boundary is greater than 3.4 MPa then elements are added at that node, and if the stress is less than 2.6 MPa, removal of all the elements which contain that node along the

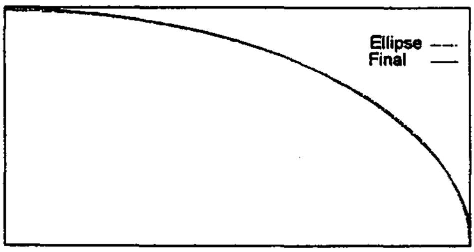


Fig. 6 Final shape of hole

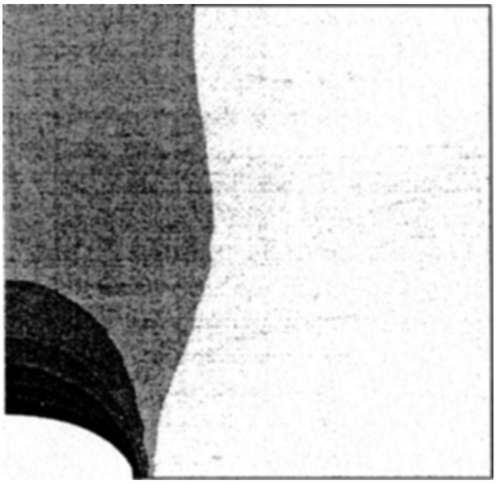


Fig. 7 von Mises stress contour after optimization

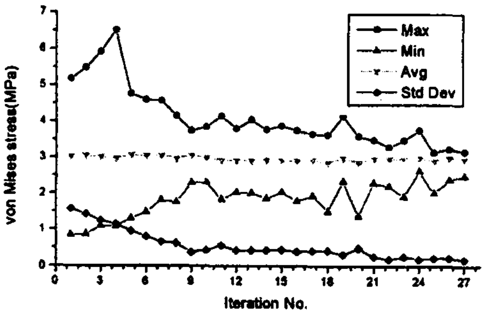


Fig. 8 The von Mises stresses along the boundary verses iteration number

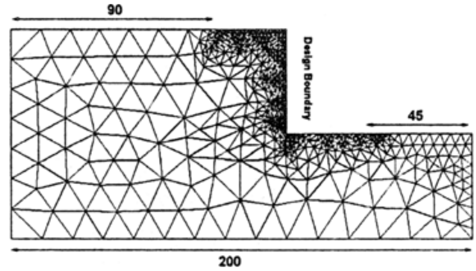


Fig. 9 Initial shape and model of a fillet (unit : mm)

boundary. A typical result of an addition and removing iteration can be seen in the Fig. 2. In the figure, the initial circular boundary has changed to new boundary with boundary element refinement. Note the smooth curve along the boundary by boundary smoothing procedure can pbe seen in the figure. In Fig. 5, going from the initial circular boundary to the final boundary curve shape during the iteration is shown. The final shape of the boundary has reached after 27 iterations and final curve is very close to the analytical curve of ellipse as shown in the Fig. 6.

The von Mises stress distribution in the plate with the optimal shape is shown in Fig. 7. As shown in the figure, the stress distribution along the boundary is in the same range and those results are same as the analytical solution.

Figure 8 shows that magnitude of the von Mises stresses along the boundary verses iteration

The maximum and minimum stresses are the two extreme values at the nodes along the boundary, and the average is average von Mises stress of the nodes. The standard deviation is calculated from the nodal stresses along the boundary for each iteration. It can be seen that the maximum and the minimum stresses go to close to the average stress for convergence. The average stress is almost constant to analytical value of 3 MPa. As the curve approaches the final optimal shape the stress distribution along the boundary curve is in a reduced stress range of stress band as the standard deviation is close to zero.

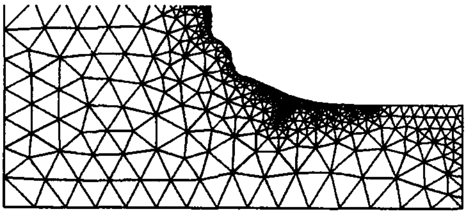
3.2 Example 2 : A fillet problem

The fillet design problem is another commonly used example of finding an optimal shape of an uniaxial tension fillet with the transition zone connecting the two different widths(Shyy and Fleury, 1988; Kim and Kwak, 1996) . A quarter of fillet is modelled due to symmetry. The initial shape and model of the fillet is as shown Fig. 9. A constant tension of 130 MPa is applied along the extreme right hand edge and symmetric displacement boundary conditions are applied on the extreme left hand edge and the bottom.

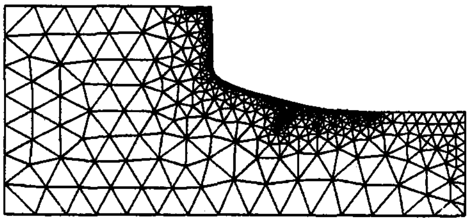
This problem does not have an unique analytical solution as in example 1. The stress distribution along the design boundary can not be constant, and the stress distribution and shape of boundary depends on the design criteria and constraints imposed on the design variables in the conventional shape optimization design.

The design boundary starts at 90 mm from right edge, and ends at 45 mm to the left of the right hand edge. The total horizontal length of the fillet is 200 mm. Along the design boundary, elements are meshed with smaller elements using the boundary refinement scheme. The shape optimize design problem is to find the boundary shape which has the minimum stress concentration. The maximum stress is at the corner with stress concentration factor of 2.31 at the initial iteration. Along the design boundary the elements are added or removed depending on the magnitude of the average element stress at the boundary nodes. In this problem elements are added if the average element nodal stress is greater than 190 Mpa, elements are removed if the average element nodal stress is less than 120 MPa. A constraint on the geometry is imposed on the vertical edge of design boundary, since the stresses along the edge is much lower than 120 MPa at the edge will disappear if there is no constraint. The constraint is that the design boundary edge can not cross the vertical line 90 mm from the left hand edge.

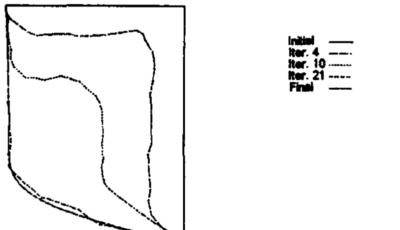
Figure 10 shows the shape of the design boundary during the iteration. One of the typical boundary and mesh shapes after iteration 15, the final shape can be seen in Fig. 10(a) and Fig. 10 (b) respectively. Figure 10(c) shows design history of the boundary from the initial shape to the final shape. The finer elements along the boundary and the smooth boundary curve can be found in Fig. 10(a) and (b). As the stress is almost zero near the top corner and highest at the lower corner region in Fig. 9, the materials are removed from top corner and added to the lower corner as in Fig. 10 (a). The final boundary shape of Fig. 10(c) converged after 29 iterations.



(a) Boundary shape of fillet after 15 iterations



(b) Final boundary shape of fillet



(c) Shape of the design boundary history

Fig. 10 The shape and mesh of the fillet during iteration

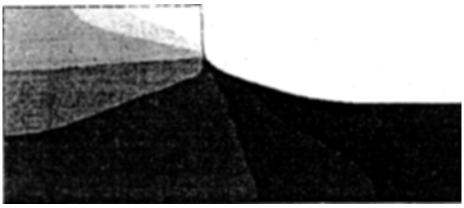


Fig. 11 Stress contours of final design

The stress contour of final design can be seen in Fig. 11. The stress in the final design is not evenly distributed since the stress along the boundary varies from a lower value close to zero to the maximum value. The maximum stress exists at the location where the narrower width begins.

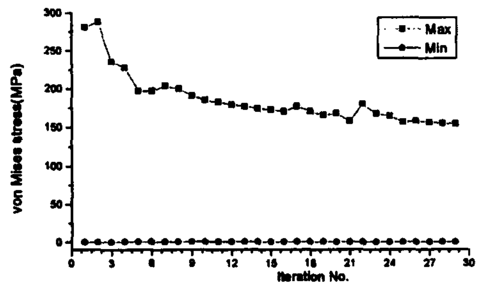


Fig. 12 The stress history during iteration

From Fig. 12, the change in the maximum and the minimum stresses can be seen. The stress concentration can be reduced almost by a half with the final boundary curve.

It can be seen that the minimum stresses remains close to zero since the minimum stress is always found at the upper corner. As the curve approaches the final optimal shape the maximum stress along the boundary curve converges to a constant value of 152 MPa.

4. Conclusions

A simple SDS based procedure to add and remove material simultaneously along the boundary is developed to optimize the shape of a two dimensional problem and applied successfully to the two example problems. The previous application of the method to structural design is focused mainly on the topology design of structures. In this paper, the shape optimization was conducted to find the exact boundary curve for a two dimensional elastic problem for minimising the maximum stress. The results shows that the results for the two dimensional infinite plate with a hole is close to the theoretical result of an elliptical boundary, and the stress concentration being reduced by a half for the fillet problem. Also the developed boundary smoothing and boundary refinement technique are successfully applied to find optimal shape of boundary.

The advantages of the method used in this paper are as follows. The most important part of the proposed method is its simplicity. It does not need to be developed into a formal optimization problem, then calculate derivatives, remeshing the whole domain. The lack of the conventional derivative based optimization enables the method to be easily understood and implemented. Because the scheme adds and removes material along the boundary node simultaneously, it performs only one finite element analysis for each iteration. This again reduces the overall computational requirements significantlly. The final stress along the boundary can be controlled by setting different values for the maximum and the minimum stress thresholds. Therefore a different optimal shape is possible for different stresses thresholds.

In summary, the proposed shape optimization method, when compared to existing derivative based shape optimization methods, has a many features such as simplicity, applicability, flexiblity, computational efficiency and much better control on stresses on the design boundary.

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